Mathematics in the Primary Program

Common Understandings

Children of different ages understand mathematical concepts in different ways. Even children of similar ages in the same classroom may be at different developmental levels. By watching children as they play with objects and interact with each other, teachers can recognize different stages of development in mathematical concepts and can plan appropriately.

Babies do not realize that a toy that falls out of the crib still exists when they cannot see it. If their rattle is out of sight, it is also "out of mind." But by the age of one year, most infants have developed a sense of object permanence (Piaget, 1954). They know their rattle is somewhere, even when they are not playing with it. This understanding is an early building block of mathematical concepts children will develop later on.

We have seen that children learn the real basics of thinking about mathematics through personal experience and playful activities. With appropriate learning experiences from birth through the early elementary years, children will develop a lifelong interest in using mathematics.

McCracken, 1987

The young child's thinking is not always consistent or logical from an adult's point of view. Many two-year-olds have begun learning their first counting words, and they may know that three is more than one or two, but it will be a while before they understand larger numbers and ideas such as eight is more than seven and less than nine. Preschoolers' understandings of number, space, time, size, and other concepts are ruled by perception and depend upon how something looks to them. They believe that ten crackers spread out in a line is more than ten crackers placed close together. Young children often think that the spread-out row has "more to eat because it looks bigger." Between six and eight years, many children begin to reason that ten things are ten things, no matter how they are grouped or arranged in space. They have developed conservation of number; that is, they are able to separate number from length, and they tend not to confuse the two concepts (Piaget & Szeminska, 1952).

Preschoolers' understanding of math concepts grows as they have many opportunities to play with objects and learn. At first, a four-year-old might try to fit a little shoe on a big doll's foot and a big shoe on a little doll's foot. Older children know the big shoe has to go with the big doll and the little shoe with the little doll. But the preschooler's way of thinking is an important step in understanding what it means to measure and will be a foundation for future mathematical learning.

Children in elementary school need many opportunities to develop math concepts through actions on concrete objects during spontaneous play, situations of daily living and projects. As children make decisions about how to arrange a group of blocks, sort, and count a set of buttons or compare objects on a balance, they begin to construct mental relationships. A child might separate a group of toy animals into two groups and then recombine the animals to make one group again. Such concrete

experiences help children to develop logical-mathematical thinking that forms the basis for understanding the meaning of abstract symbols such as 3 + 5 = 8.

Learning Through Mathematics

Mathematics exploration in the primary years should be related to the child's immediate environment and should always be based on a sound foundation of concrete experiences. The classroom should contain many sets of objects for counting, matching, classifying, ordering, and making spatial relations. Activities can provide noncompetitive practice for concepts which have previously been developed.

Whenever new material and concepts are introduced, children at all stages of the primary program require extended periods of time to freely explore materials and concepts before more formal instruction begins. The exploration stage is the major focus of mathematical experiences for children at the beginning of the primary program (Baratta-Lorton, 1995).

Math activities are integrated with other relevant science or social studies projects such as plotting the growth of baby hamsters or making a model of the neighborhood.

Teachers create a risk-free environment that invites inquiry. They guide children's investigations by observing, asking questions, and making open suggestions that stimulate the development of logical thinking. They realize that all the steps children go through in thinking are important and children's "mistakes" are a necessary part of learning. They encourage children to self-correct and evaluate their own progress.

The goal of the math program is to enable children to use math through exploration, discovery, and solving meaningful problems. It is through appropriate experiences presented in logical sequences that positive dispositions develop and effective learning occurs. As children create and solve problems, they become confident in their own ability to make sense of their world and accept new mathematical challenges. While some aspects of mathematics will be integrated through themes and topics of study it is appropriate to introduce and practice some mathematical concepts and skills in specific blocks of instructional time devoted to mathematics.

Considerations for Teaching Mathematics

- Math has to do with the relationships among objects, events, and people, such as "how many," "how much," "larger than," "smaller than," "same," and "different." Young children explore these relationships through classifying, ordering, number, measurement, space, and time.

- Children within a group will be at different levels of development with regard to mathematical concepts. Mathematics concepts must be introduced in an age appropriate and individually appropriate manner.

- Children develop math concepts naturally through many experiences with concrete objects throughout the primary years before they can understand math concepts in an abstract way. They make a gradual transition to mathematical symbols through describing, drawing pictures, and writing about their experiences. Children become confused and frustrated when rushed into symbols too soon.

- Math concepts are used every day. Understanding math concepts helps people function in the world. This occurs through the daily use of common objects in the child's world.

- Teachers must decide and agree upon when it is generally appropriate to introduce math concepts. Teachers can gain valuable insights by doing their own informal research with children. Along with observations, readings and discussions about developmental stages in children's understanding of mathematics help teachers form the basis for curriculum decisions.

- Assessments in mathematics inform teachers' decision-making process. They should be stress-free and often provide valuable learning experiences for the child. Assessments may take the form of observations, tasks and interviews, and talking with children about their drawings, stories or other work. During the primary years when children are developing concepts, timed testing is not appropriate.

Children and Mathematics: Implications for the Primary Curriculum

An appropriate curriculum for young children to meet the NCTM Standards' overall goals must do the following:

1. *Address the relationship between young children and mathematics*. Children enter kindergarten with considerable mathematical experience, a partial understanding of many concepts, and some important skills, including counting. Nonetheless, it takes careful planning to create a curriculum that capitalizes on children's intuitive insights and language in selecting and teaching mathematical ideas and skills. It is clear that children's intellectual, social, and emotional development should guide the kind of mathematical experiences they should have in light of the overall goals for learning mathematics. The notion of a *developmentally appropriate* curriculum is an important one.

A developmentally appropriate curriculum encourages the exploration of a wide variety of mathematical ideas in such a way that children retain their enjoyment of and curiosity about mathematics. It incorporates real world contexts, children's experiences, and children's language in developing ideas. It recognizes that children need considerable time to construct sound understandings and develop the ability to reason and communicate mathematically. It looks beyond what children appear to know to determine how they think about ideas. It provides repeated contact with important ideas in varying contexts throughout the year and from year to year.

Programs that provide limited developmental work, that emphasize symbol manipulation and computational rules, and that rely heavily on paper-and-pencil work sheets do not fit the natural learning patterns of children and do not contribute to important aspects of children's mathematical development.

2. *Recognize the importance of the qualitative dimensions of children's learning*. The mathematical ideas that primary level children acquire form the basis for all further study of mathematics. Although quantitative considerations have frequently dominated discussions in recent years, qualitative considerations have greater significance. Thus, how well children come to understand mathematical ideas is far more important than how many skills they acquire. The success with which programs at later grade levels achieve their goals depends largely on the quality of the foundation established during the first five years of school.

3. *Builds beliefs about what mathematics is, about what it means to know and do mathematics, and about children's view of themselves as mathematics learners*. The beliefs that young children form influence not only their thinking and performance during this time but also their dispositions and decisions about studying mathematics in later years. Beliefs also become more resistant to change as children grow older. Thus, affective dimensions of learning play a significant role in and must influence curriculum and instruction.

Assumptions

Several basic assumptions governed the selection and shaping of the NCTM K-4 Standards.

1. The primary curriculum should be conceptually oriented. The view that the primary curriculum should emphasize the development of mathematical understandings and relationships is reflected in the discussions about the content and emphasis of the curriculum. A conceptual approach enables children to acquire clear and stable concepts by constructing meanings in the context of physical situations and allows mathematical abstractions to emerge from empirical experience. A strong

conceptual framework also provides an anchor for skill acquisition. Skills can be acquired in ways that make sense to children and in ways that result in more effective learning. A strong emphasis on mathematical concepts and understandings also supports the development of problem solving.

Emphasizing mathematical concepts and relationships means devoting substantial time to the development of understandings. It means relating this knowledge to the learning of skills by establishing relationships between the conceptual and procedural aspects of tasks. The time required to build an adequate conceptual base should cause educators to rethink when children are expected to demonstrate a mastery of complex skills. A conceptually oriented curriculum is consistent with the overall curricular goals in this report and can result in programs that are better balanced, more dynamic, and more appropriate to the intellectual needs and abilities of children.

2. *The primary curriculum should actively involve children in doing mathematics*. Young children are active individuals who construct, modify, and integrate ideas by interacting with the physical world, materials, and other children. Given these facts, it is clear that the learning of mathematics must be an active process. Throughout the NCTM Standards, such verbs as *explore, justify, represent, solve, construct, discuss, use, investigate, describe, develop,* and *predict* are used to convey this active physical and mental involvement of children in learning the content of the curriculum.

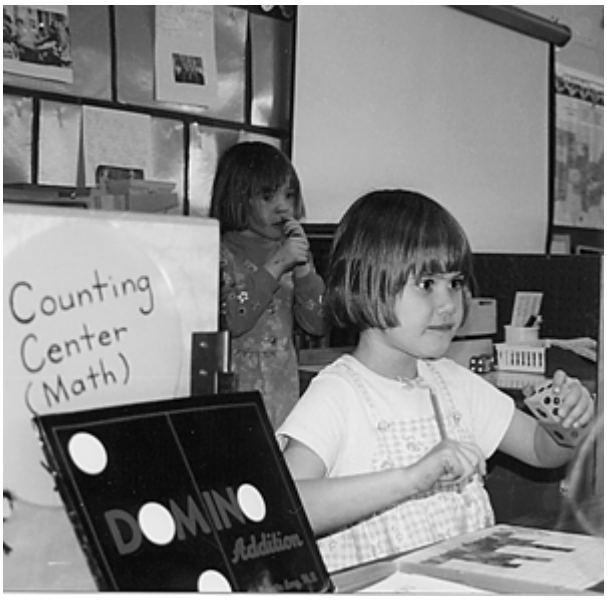
The importance of active learning by children has many implications for mathematics education. Teachers need to create an environment that encourages children to explore, develop, test, discuss, and apply ideas. They need to listen carefully to children and to guide the development of their ideas. They need to make extensive and thoughtful use of physical materials to foster the learning of abstract ideas.

Primary classrooms need to be equipped with a wide variety of physical materials and supplies. Classrooms should have ample quantities of such materials as counters; interlocking cubes; connecting links; base-ten, attribute, and pattern blocks; tiles; geometric models; rulers; spinners; colored rods; geoboards; balances; fraction pieces; and graph, grid, and dot paper. Simple household objects, such as buttons, dried beans, shells, egg cartons, and milk cartons also can be used.

3. *The primary curriculum should emphasize the development of children's mathematical thinking and reasoning abilities*. An individual's future uses and needs for mathematics make the ability to think, reason, and solve problems a primary goal for the study of mathematics. Thus, the curriculum must take seriously the goal of instilling in students a sense of confidence in their

ability to think and communicate mathematically, to solve problems, to demonstrate flexibility in working with mathematical ideas and problems, to make appropriate decisions in selecting strategies and techniques, to recognize familiar mathematical structures in unfamiliar settings, to detect patterns, and to analyze data. The NCTM Standards reflect the view that mathematics instruction should promote these abilities so that students understand that knowledge is empowering and that individual pieces of content are all related to this broader perspective.

Developing these characteristics in children requires that schools build appropriate reasoning and problem solving experiences into the curriculum from the outset. Further, this goals needs to influence the way mathematics is taught and the way students encounter and apply mathematics throughout their education.



4. *The primary curriculum should emphasize the application of mathematics*. If children are to view mathematics as a practical, useful subject, they must understand that it can be applied to a wide variety of real world problems and phenomena. Even though most mathematical ideas in the primary curriculum arise *from* the everyday world, they must be regularly applied *to* real world situations. Children also need to understand that mathematics is an integral part of real world situations and activities in other curricular areas. The mathematical aspects of that work should be highlighted.

Learning mathematics has a purpose. At the primary level, one major purpose is helping children understand and interpret their world and solve problems that occur in it. Children learn computation to solve problems; they learn to measure because measurement helps them answer questions about how much, how big, how long, and so on; and they learn to collect and organize data because doing so permits them to answer other questions. By applying mathematics, they learn to appreciate the power of mathematics.

5. *The primary curriculum should include a broad range of content*. To become mathematically literate, students must know more than arithmetic. They must possess a knowledge of such important branches of mathematics as measurement, geometry, data analysis, probability, and algebra. These increasingly important and useful branches of mathematics have significant and growing applications in many disciplines and occupations.

The curriculum at all levels needs to place substantial emphasis on these branches of mathematics. Mathematical ideas grow and expand as children work with them throughout the curriculum. The informal approach at this level establishes the foundation for further study and permits children to acquire additional knowledge they will need. These topics are highly

appropriate for young learners because they make important contributions to children's mathematical development and help them see the usefulness of mathematics. They also provide productive, intriguing activities and applications.

The inclusion of a broad range of content in the curriculum also allows children to see the interrelated nature of mathematical knowledge. When teachers take advantage of the opportunity to relate one mathematical idea to others and to other areas of the curriculum, children acquire broader notions about the interconnectedness of mathematics and its relationships to other fields. The curriculum should enable all children to do a substantial amount of work in each of these topics at each grade level.

6. *The primary curriculum should make appropriate and ongoing use of calculators and computers*. Calculators can be a valuable tool for learning mathematics. Calculators enable children to explore number ideas and patterns, to focus on problem solving processes, and to investigate realistic applications. The thoughtful use of calculators can increase the quality of the curriculum as well as the quality of children's learning.

Teachers must be aware of children's developmental levels before introducing calculators. The child who does not conserve numbers and who has not constructed an understanding of the operations of addition and subtraction will not benefit from using a calculator to add and subtract until the logic to support its use has been developed. Calculators do not replace the need for children to develop these basic arithmetic understandings, to compute mentally, or to do reasonable paper-and-pencil computation. Classroom experiences indicate that young children take a common sense view about calculators and recognize the importance of not relying on them when it is more appropriate to compute in other ways. The availability of calculators means, however, that educators must develop a broader view of the various ways computation can be carried out and must place less emphasis on complex paper-and-pencil computation. Calculators also highlight the importance of teaching children to recognize whether results are reasonable.

The power of computers also needs to be used in contemporary mathematics programs. Computers cannot replace the child's need for actions on concrete objects, but once a child has developed a particular reasoning process, there are some excellent programs to help children rehearse, review, and extend concepts.

The thoughtful and creative use of technology can greatly improve both the quality of the curriculum and the quality of children's learning. Integrating calculators and computers into

school mathematics programs at appropriate levels is critical in meeting the goals of redefining curriculum.

Mathematics Processes in the Primary Program

Mathematics as Problem Solving

Problem solving should be the central focus of the mathematics curriculum, a primary goal of all mathematics instruction and an integral part of every activity. Problem solving should permeate the entire program and provide the context in which concepts and skills are developed. A comprehensive and rich classroom environment supports and encourages problem solving efforts so that children feel free to share their thinking, take risks, try different strategies and ways of representing problems and value the problem solving process as much as the solution.

Problem solving in this sense is not the typical "story problem" at the end of a chapter in a math book. Rather, most problem situations arise naturally from school and everyday experiences that have meaning to children. In this context, problems may be thought of as "challenges," "speculations," "investigations" or "explorations" not only with numbers, but in all areas of mathematical thinking. The teacher may pose problems in the form of thought-provoking questions such as, "What do you think would happen if…?" or "Do you think there is any way to…?" Better yet, children are encouraged to create their own problems which have greater relevance for them.

Children are not directly taught steps of problem solving. Rather, through each child's unique problem solving process, they develop their own strategies such as using manipulatives, trial and error, drawing a picture, looking for a pattern or acting out a problem.

In the primary program, the child is provided numerous opportunities for problem solving to:

- Use problem solving approaches to investigate and understand mathematical content
- Formulate problems from everyday and mathematical situations
- Develop and apply strategies to solve a wide variety of problems
- Verify and interpret results with respect to the original problem
- Acquire confidence in using mathematics meaningfully

Mathematics as Reasoning and Proof

A major goal of mathematics instruction is to help children develop the belief that they have the power to do mathematics and that they have control over their own successes or failures. This autonomy develops as children gain confidence in their ability to reason and justify their thinking. It grows as children learn that mathematics is not simply memorizing rules and procedures, but that mathematics makes sense, is logical, and is enjoyable (NCTM, 2000).

Children develop mathematical reasoning thinking by acting on objects and by reflecting upon their actions. For example, a child classifies a set of attribute blocks into groups. As the child decides how to group or order the blocks, he/she is developing the logic of classes. In real-life problem solving situations such as creating a board game, the child must consider different possible moves and outcomes. Reasoning is also involved in the construction of various physical quantities such as conservation of amount (when water is poured from a tall, thin bottle into a short, fat bottle the child reasons it is the same amount because none was added or taken away).

At the primary level, the child is provided with numerous opportunities for reasoning in order to:

- Draw logical conclusions about mathematics
- Use manipulatives, models, known facts, properties, and relationships to explain their thinking
- Justify their answers and solution processes
- Use patterns and relationships to analyze mathematical situations
- Believe that mathematics makes sense (NCTM, 2000)

The child is provided with opportunities for developing mathematical reasoning, through activities such as:

- Exploring relationships among attribute blocks
- Exploring, creating and extending patterns
- Creating riddles (*What Am I?* I have 3 or 4 sides. All my angles are equal. My sides are not all equal. *Who Am l?* I am more than 20 and less than 30. I am not 25.)
- Developing and testing conjectures (looking for patterns using a l00's chart)
- Describing thinking strategies for the solution to a problem (think aloud)

Mathematics as Communication

Communication plays an important role in helping children construct links between their informal, intuitive notions and the abstract language and symbolism of mathematics. It also plays a key role in helping children make important connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas (NCTM, 2000). Attending to students' communications about their thinking provides teachers with a rich information base from which they can make sound instructional decisions.

Communication connections can be made between mathematics and literature. There are many fine children's books that contain math-related ideas such as counting, problem solving, money, seasonal cycles, time sequencing, relationships, etc. After hearing and reading stories, children can be encouraged to act out the stories with manipulatives or puppets, create solutions to problems found in books, and write and illustrate their own books. In the primary program, the child is provided with opportunities for communication to:

- Relate physical materials, pictures, and diagrams to mathematical ideas
- Reflect on and clarify their thinking about mathematical ideas and situations
- Relate their everyday language to mathematical language and symbols
- Realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics (NCTM, 2000)

The child is provided with opportunities for developing mathematics communication through activities such as:

- Keeping a mathematics journal
- Writing a letter/list
- Role playing (sales clerk and shopper in a store)
- Collaborating on a project (deciding how large a cage to build for the class pet rabbit)
- Creating and illustrating a book about mathematics

Mathematical Connections

As children construct their understanding of mathematics, it is important that they connect their own intuitive knowledge with new ideas. When mathematical ideas are also connected to everyday experiences, both in and out of school, children become aware of the usefulness of mathematics (NCTM, 2000). Connections also need to be made within and among the various topics of mathematics.

At the primary level, the child is provided with numerous opportunities to make connections to:

- Link concepts through active learning
- Relate various representations of concepts to one another
- Recognize relationships among different topics in mathematics
- Use mathematics in other curriculum areas
- Use mathematics in their daily lives (NCTM, 2000)

At the primary level, the child is provided with opportunities for developing mathematical connections through:

- Providing extended exposure to integrated topics through projects and thematic units
- Using mathematical applications such as measurement throughout the year
- Looking for and integrating mathematical connections with other subject areas such as science, art, and the language arts
- Letting mathematical ideas naturally flow from one lesson to another, allowing time for students to explore, discuss, and generalize mathematical connections
- Encouraging students to compare and contrast

Content Strands

Number Sense and Numeration

Children must understand numbers if they are to make sense of the ways numbers are used in the everyday world. Intuition about number relationships helps children make judgments about the reasonableness of estimates and computational results. Children with good number sense have well-understood number meanings, have developed multiple relationships among numbers, and understand the effects of operations on numbers.

Children construct number meanings gradually through manipulating physical objects and using their own language to explain their thinking. Children's experiences with numbers are most

beneficial when the numbers have meaning for them. Number symbols should be linked to concrete materials, and symbols should only be introduced after the child has had sufficient time to construct meaning with objects. More advanced concepts such as place value are not introduced until the child has a good grasp of relationships among lower numbers. Larger numbers should not be introduced in isolation; rather children should have the opportunity to "build" them by arranging objects in groups, adding, and counting.

In the primary program, the child is provided with numerous opportunities to use whole number concepts and skills in order to:

- Construct number meanings through real world experiences and the use of physical materials
- Count, match, order, group, and classify
- Develop number sense
- Develop an awareness of numbers encountered in the real world

Estimation

Estimation presents students with another dimension of mathematics. There are many instances in real life when an exact count or measure is either not necessary or impossible. When we use terms such as "about," "close to," or "a little less than," we are estimating.

Estimation should be an ongoing part of children's experiences. Children can be encouraged to develop an "estimation mindset" that includes what is meant by "estimate," when estimation is appropriate and how close an estimate is required in different situations. If they are encouraged to estimate, they will accept estimation as a legitimate part of mathematics.

When children enter school, they already have intuitive estimation abilities. For example, they know they are "almost six years old" or when it is "about noon." Yet when asked to estimate large numbers or abstract quantities out of their realm of experience, young children may give wildly divergent answers. The child who does not yet conserve number should not be asked to estimate how many seeds in a pumpkin. A suggestion is to begin estimation activities with small quantities of concrete objects and work up to larger numbers as children's mathematical abilities develop. Children are encouraged to estimate and then verify their estimate through actually counting. In this way, their estimates become more logical and reasonable.

In the primary program, the child is provided with numerous opportunities to estimate in order to:

- Explore estimation strategies
- Recognize when an estimate is appropriate
- Determine the reasonableness of results
- Apply estimation in working with quantities, measurement, computation and problem solving

The child is provided with opportunities for developing estimation through activities involving:

- Estimating quantity (and verifying by counting)
- Estimating length (and verifying by measuring)
- Estimating computation
- Estimating as a check when using calculators

Concepts of Whole Number Operations

Understanding the fundamental operations of addition, subtraction, multiplication, and division is central to knowing mathematics. These operations have their genesis in physical actions on objects. The young child is already intuitively familiar with combining and separating sets of objects. Many children have had experiences such as sharing a package of cookies among three friends.

Children need to develop "operation sense." Children develop concepts and relationships as they encounter the four basic operations in a wide variety of problem structures. Children with operation sense understand that addition and subtraction are opposite actions. A child might add 3 frogs to 4 toads to make 7 and then make the frogs jump away leaving only 4 toads. In addition to problems involving joining and separating, teachers should suggest problems involving comparing and equalizing. For example, a child might be given a situation in which one dog has 5 bones and the other has 9 with the question, "Can you make it fair?"

Gradually, as the child demonstrates understanding of the meaning of an operation, the symbols (+, -, x, ÷) can be introduced.

In the primary program, the child is provided with numerous opportunities to use addition, subtraction, multiplication, and division of whole numbers in order to:

- Develop meaning for the operations by acting on objects and discussing a rich variety of problem situations
- Relate the mathematics language and symbolism of operations to real problem situations and informal language
- Recognize that a wide variety of problem structures can be represented by a single operation
- Develop operation sense.

Whole Number Computation

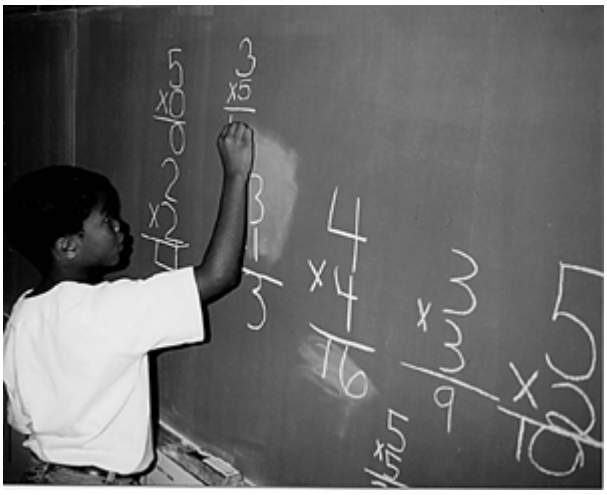
The purpose of computation is to solve problems. Although computation is important in mathematics and in daily life, our technological age requires us to rethink how computation is done today. Almost all complex computation today is done by calculators and computers. In many daily situations, answers are computed mentally or estimates are sufficient. There are also times when paper-and-pencil algorithms are useful. It is important for children to know a variety of methods of computation and for teachers to have reasonable expectations regarding proficiency.

Clearly, paper-and-pencil computation cannot continue to dominate the curriculum or there will be insufficient time for children to learn other, more important mathematics they need now and in the future. Isolated paper-and-pencil drill and premature expectations of mastery are inconsistent with the NCTM Standards, cause poor retention, and require large amounts of time for re-teaching. By

emphasizing underlying concepts, having children use physical materials, linking manipulation of objects to steps of the procedures, and developing thinking patterns, teachers can help children develop knowledge of basic facts and algorithms in a meaningful way. Rather than rote memorizing nonsense, children can construct computational facts using objects. Research indicates the conceptual approach results in better retention and, because children will not have to relearn what they memorized and forgot, the amount of time to learn computation is reduced in the long run.

In the primary program, the child is provided with numerous opportunities to develop whole number computation in order to:

- Model, explain, and develop reasonable proficiency with basic facts in application settings using a variety of manipulative objects
- Use a variety of mental computation and estimation techniques
- Use calculators in appropriate computational situations
- Select and use computation techniques appropriate to specific problems and determine whether the results are reasonable



Common Fractions

Fractions represent an extension of the child's knowledge about number. Experiences that children have in the early levels form a foundation for symbolic work with fractions in the upper grades.

All work with fractions at the primary level should involve situations of everyday life that can be easily modeled with real objects. Children have usually had experiences with sharing jellybeans or dividing a candy bar among friends. Subdivision of a whole into equal parts is fundamental to understanding fractions. Children can engage in many types of activities to develop an understanding of subdivision, such as folding and cutting paper strips.

Symbols are introduced only after children have developed the concepts and oral language necessary for symbols to be meaningful and should be connected to concrete objects and to oral language.

In the primary program, the child is provided with numerous opportunities to use fractions to:

- Develop concepts of common fractions using manipulatives and real materials
- Develop number sense for common fractions
- Use models to explore equivalent fractions
- Apply fractions to problem situations

Patterns and Relationships (Algebra)

Patterns are everywhere. Exploring, identifying, and working with a wide variety of patterns help children to develop the ability to classify and organize information, and to understand how mathematics applies to the world in which they live. Working with patterns fosters the kind of mathematical thinking that serves as a foundation for the more abstract ideas studied in later years.

From the earliest years, children should be encouraged to look for patterns and regularities in events, shapes, designs, and numbers. Physical materials should be used to help children recognize and create patterns of their own. From this intuitive beginning, children later generalize pattern to the entire mathematical system.
In the primary program, the child is provided with numerous opportunities for the study of patterns and relationships in order to:

- Recognize, describe, extend, and create a wide variety of patterns, (taking a "pattern walk in the neighborhood," or making a placemat for a gift using rubber stampers to create a pattern around the border; stringing colored macaroni in patterns)
- Represent and describe mathematical relationships

Geometry and Spatial Sense

Geometry helps us represent and describe in an orderly manner the world in which we live. Children are naturally interested in the spatial activities and find them intriguing and motivating. Some of the first relationships that children build are spatial, concepts such as inside, outside, next to, over, and under. Since their early spatial abilities frequently exceed numerical skills, tapping on these strengths forms a basis for the development of other mathematical ideas.

Spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world. Children who develop a strong spatial sense are better prepared to develop concepts of number and measurement.

In developing geometric understandings, children need to investigate, experiment, and explore with everyday objects and other physical materials. As children work with a variety of objects such as blocks and geoboards, they learn about the properties of shapes. Folding and cutting paper shapes and using mirrors develops concepts of symmetry. Activities that ask children to visualize, draw, and compare objects in various positions help develop spatial sense.

Language (terminology, shape names) should not be the focus but should grow naturally out of experiences with objects. Of particular importance is having children draw from real objects, because this helps children construct spatial relationships. Drawing should be part of learning activities each day.

In the primary program, the child is provided with numerous opportunities to use two and three-dimensional geometry in order to:

- Describe, model, draw, and classify shapes
- Investigate and predict the results of combining, subdividing, and changing shapes
- Develop spatial sense by constructing arrangements in space; drawing objects in various positions; observing effects of rotations and displacements; and imagining how objects would look from different points of view
- Relate geometric ideas to number and measurement ideas
- Recognize and appreciate geometry in the world.

Measurement

Measurement is important because of its usefulness in every day life. At the primary level emphasis is placed on developing a foundation in the basic concepts underlying measurement. Children need to understand the attribute being measured as well as what it means to measure. Before they are capable of such understanding, they must first experience a variety of activities that focus on comparing objects directly, covering them with various units, and counting the units.

If children's initial explorations use nonstandard units, they will develop some understandings about units and come to recognize the necessity of standard units in order to communicate. This process, however, cannot be rushed. Premature use of instruments (such as rulers) and formulas leaves children without the understanding necessary to solve measurement problems. If a child does not yet conserve length (thinks that a stick gets longer or shorter depending upon how it looks), what sense can measuring with a ruler make?

Estimation activities should be integrated throughout measurement. The child should first estimate and then verify with real objects. Textbook exercises cannot substitute for activities that answer meaningful questions about real problems.

In the primary program, the child is provided with numerous opportunities to use measurement in order to:

- Understand the attributes of length, capacity, mass (weight), area, time, and temperature
- Develop the process of measuring and concepts related to units of measurement
- Make and use estimates of measurement
- Make and use measurements in every day situations.

Data Analysis

Collecting, organizing, describing, displaying, and interpreting data, as well as making decisions and predictions on the basis of that information are important skills in our society. These processes are particularly appropriate for young children because they can be used to solve problems that are inherently interesting and offer rich opportunities for mathematical inquiry.

Graphing is a natural outgrowth of many activities. As children arrange and group objects, they often spontaneously line them up in one-to-one correspondences. A next step would be to represent the objects with pictures and later symbols. Young children should be encouraged to construct their own graphic representations of objects rather than being given adult formats to fill in.

Children are very interested in information about themselves. An appropriate group activity is the collection of data such as, “What are our favorite ice cream flavors?” “What pets do we own?” or any number of characteristics such as eye color or gender. Class graphs can give children a sense of the group characteristics as well as experience with representing data.

In the primary program, mathematics should include experiences with data analysis and probability so students can:

- Collect, organize, and describe data
- Construct, read, and interpret displays of data (such as graphs)
- Formulate and solve problems that involve collecting and analyzing data

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Descriptors of Learning in Mathematics

Early Primary	Later Primary
Dispositions	
The child: ▪ Develops confidence in using mathematics meaningfully ▪ Recognizes and appreciates mathematics in the world ▪ Applies mathematics to everyday problems ▪ Enjoys participating in mathematics activities	The child: ▪ Develops confidence in using mathematics meaningfully ▪ Recognizes and appreciates mathematics in the world ▪ Applies mathematics to everyday problems ▪ Enjoys participating in mathematics activities
Mathematics Processes	
Mathematics as Problem Solving	
The child: ▪ Formulates problems from everyday and mathematical situations ▪ Recognizes and restates problem using objects, pictures, or words ▪ Clarifies problem by asking questions ▪ Solves problems by acting out, making a diagram or constructing a model	The child: ▪ Formulates problems from everyday and mathematical situations ▪ Recognizes and restates problem using objects, pictures, or words, and number sentences ▪ Clarifies problems by exploring alternate interpretations ▪ Solves problems by making a pattern, guess and check, or by writing and solving number sentences ▪ Verifies and interprets results of problem solving ▪ Uses a calculator (where appropriate) to solve problems

Descriptors of Learning in Mathematics

Early Primary	Later Primary
Mathematics Processes	
Mathematics as Reasoning and Proof	
The child: ▪ Demonstrates confidence in ability to reason and justify thinking ▪ Sees logic and believes that mathematics makes sense	The child: ▪ Demonstrates confidence in ability to reason and justify thinking ▪ Sees logic and believes that mathematics makes sense
Mathematics as Communication	
The child: ▪ Relates concrete materials to mathematical ideas ▪ Relates pictures and diagrams to mathematical ideas ▪ Relates everyday language to mathematical language and symbols	The child: ▪ Makes connections among concrete, pictorial, and symbolic abstract representations of mathematical ideas ▪ Relates pictures and diagrams to mathematical ideas ▪ Relates everyday language to mathematical language and symbols ▪ Reflects on and clarifies thinking about mathematical ideas
Mathematics as Connections	
The child: ▪ Connects own knowledge with new ideas ▪ Uses mathematics in daily life ▪ Uses mathematics in other curriculum areas	The child: ▪ Connects own knowledge with new ideas ▪ Uses mathematics in daily life ▪ Uses mathematics in other curriculum areas ▪ Recognizes relationships among different topics in mathematics

Descriptors of Learning in Mathematics

Early Primary	Later Primary
Content Strands	
Number Sense and Numeration	
The child: ▪ Develops one-to-one correspondence among sets of objects, creates equivalent groups ▪ Relates a single numeral to a group of objects, uses number words ▪ Counts forward and backward ▪ Uses ordinal numbers ▪ Compares groups using “more,” “fewer” ▪ Estimates number of checks by counting and grouping ▪ Applies the use of number to everyday situations	The child: ▪ Counts larger numbers, skip counts ▪ Relates numerals to numbers of objects ▪ Realizes that a given number of objects remains constant regardless of arrangement (conserves number) ▪ Demonstrates understanding of the numeration system ▪ Refines estimates and checks by counting and grouping ▪ Groups objects by fives, tens ▪ Develops concept of place value (including regrouping) using concrete materials and symbols.
Estimation	
The child: Explores estimation strategies Estimates small numbers of objects and verifies by counting Uses estimation in problem solving	The child: Explores estimation strategies Recognizes when an estimate is appropriate Uses estimation to check computation Determines reasonableness of results Applies estimation in working with quantities, measurements, computation, and problem solving

Descriptors of Learning in Mathematics

Early Primary	Later Primary
Content Strands	
Concepts of Whole Number Operations and Computation	
The child: ▪ Demonstrates processes of addition and subtraction by combining and separating objects ▪ Explores possible combinations for a given number of objects ▪ Records combinations by drawing, stamping or pasting objects ▪ Solves simple verbal problems involving addition and subtraction with objects	The child: ▪ Demonstrates processes of addition and subtraction by combining and separating objects ▪ Constructs and demonstrates knowledge of combinations for numbers (to ten and beyond) ▪ Records combinations by drawing and writing stories ▪ Develops understanding of terminology (add, take away, plus, minus, equals) in problem solving contests ▪ Reads and writes expressions and number sentences in horizontal and vertical formats ▪ Creates and solves addition and subtraction problems of various structures (sums, how many more, how many left, difference, multi-step problems) ▪ Creates, solves, writes, and illustrates own story problems ▪ Finds and records sum of several one-digit numbers ▪ Using objects, finds and records sum of two-digit numbers with and without grouping ▪ Explores multiplication and division by creating arrays, sharing, and repeated addition of objects ▪ Records multiplication and division of single digits by drawing and writing

Descriptors of Learning in Mathematics

Early Primary	Later Primary
Content Strands	
Common Fractions	
The child: ▪ Develops understanding of sharing by dividing whole objects or groups of objects into equal-sized amounts	The child: ▪ Relates the language of sharing whole objects or groups of objects to common fractions ▪ Recognizes and represents common fractions using objects and by drawing ▪ Compares common fractions using objects or by drawing
Patterns and Relationships	
Patterns The child: ▪ Recognizes, describes, and creates patterns ▪ Reverses, extends, and inserts objects into patterns	The child: ▪ Recognizes patterns in real life and in mathematics (geometric patterns) ▪ Identifies, describes, extends and creates patterns with numbers
Relations The child: ▪ Compares objects according to size (larger, smaller) ▪ Orders objects according to magnitude of characteristic (length, size, amount)	The child: ▪ Inserts elements into the appropriate position in an ordered series ▪ Makes correspondences between ordered series and relates elements to ordinal number ▪ Orders objects by mass (using a balance); by internal volume (by pouring)

Descriptors of Learning in Mathematics

Early Primary	Later Primary
Content Strands	
Patterns and Relationships continued	
Classification The child: ▪ Makes arrangements of objects ▪ Recognizes a common characteristic in a group of objects ▪ Groups by single attribute and describes criteria	The child: ▪ Classifies objects by several attributes; maintains consistent criteria ▪ Develops class-inclusion (part-whole relationships among classes)
Geometry and Spatial Sense	
The child: ▪ Builds, describes, and draws spatial arrangements of objects ▪ Sees part-whole relationships in whole objects among groups of objects ▪ Matches two-dimensional arrangement ▪ Identifies, describes, models, draws, and classifies plane figures ▪ Classifies 3-D shapes informally ▪ Explores and develops spatial relationships such as inside/outside, top/bottom, above/below, between, next to, etc. ▪ Develops relationships of order in space ▪ Begins to draw overlapping objects ▪ Recognizes geometry in the world	The child: ▪ Identifies, describes, models, draws, and classifies polygons ▪ Identifies, describes common 3-D shapes ▪ Classifies 3-D shapes ▪ Sees relationship between plane figures and 3-D shapes ▪ Completes and creates symmetrical figures ▪ Constructs right and left relationships ▪ Graphs on a horizontal or vertical number line ▪ Knows how things would look from another point of view; begins to draw objects in perspective ▪ Knows an object does not change in length when its position changes (conserves length)

Descriptors of Learning in Mathematics

Early Primary	Later Primary
Content Strands	
Measurement	
The child: ▪ Compares lengths and sizes of objects directly ▪ Compares objects on a balance ▪ Estimates, measures, and records length using non-standard units ▪ Compares amounts of liquids using various sizes of containers ▪ Describes temperature using relative terms ▪ Explores the values of coins and uses of money in concrete problem solving situations ▪ Relates concepts of time to experience ▪ Sequences events according to time; e.g. the daily schedule	The child: ▪ Develops understanding of the need for using standard units ▪ Estimates, measures, and records length using standard units ▪ Estimates, measures, and records area using concrete materials ▪ Estimates, measures, and records mass of objects using non-standard units ▪ Investigates, estimates, and measures capacity using concrete materials ▪ Describes, reads, and records temperature ▪ Makes change; solves problems involving money ▪ Creates and solves problems involving length, area, mass or capacity ▪ Tells time using a clock
Data Analysis	
The child: ▪ Collects first-hand data by counting ▪ Sorts, classifies, and displays information using graphs (concrete, pictorial) ▪ Reads, discusses, and interprets displayed data	The child: ▪ Collects first-hand data by counting and measuring ▪ Extracts second-hand data from presented sources, (tables, lists, visuals, printed resources…) ▪ Sorts, classifies, and displays information using graphs (pictorial and symbolic) ▪ Reads, discusses, and interprets displayed data ▪ Solves problems involving diagrams, tables, graphs

NCTM Standards

(Charts from Principles and Standards for School Mathematics, 2000, reprinted with permission)

Number and Operations		
Standard Instructional programs from pre-kindergarten through grade 12 should enable all students to—	**Pre-K–2 Expectations** In pre-kindergarten through grade 2 all students should—	**Grades 3–5 Expectations** In grades 3–5 all students should—
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	▪ Count with understanding and recognize "how many" in sets of objects ▪ Use multiple models to develop initial understandings of place value and base-ten number system ▪ Develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections ▪ Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers ▪ Connect number words and numerals to the quantities they represent, using various physical models and representations ▪ Understand and represent commonly used fractions, such as, 1/3, 1/4, and 1/2	▪ Understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals ▪ Recognize equivalent representations for the same number and generate them by decomposing and composing numbers ▪ Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers ▪ Use models, benchmarks, and equivalent forms to judge the size of fractions ▪ Recognize and generate equivalent forms of commonly used fractions, decimals, and percents ▪ Explore numbers less than 0 by extending the number line and through familiar applications ▪ Describe classes of numbers according to characteristics such as the nature of their factors
Understand meanings of operations and how they relate to one another	▪ Understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations ▪ Understand the effects of adding and subtracting whole numbers ▪ Understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally	▪ Understand various meanings of multiplication and division ▪ Understand the effects of multiply and dividing whole numbers ▪ Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems ▪ Understand and use properties of operations, such as the distributivity of multiplication over addition
Compute fluently and make reasonable estimates	▪ Develop and use strategies for whole-number computations, with a focus on addition and subtraction ▪ Develop fluency with basic number combinations for addition and subtraction ▪ Use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators	▪ Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 x 50 ▪ Develop fluency in adding, subtracting, multiplying, and dividing whole numbers ▪ Develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results ▪ Develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience ▪ Use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals ▪ Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tool

Algebra		
Standard Instructional programs from pre-kindergarten through grade 12 should enable all students to—	**Pre-K–2 Expectations** In pre-kindergarten through grade 2 all students should—	**Grades 3–5 Expectations** In grades 3–5 all students should—
Understand patterns, relations, and functions	▪ Sort, classify, and order objects by size, number, and other properties ▪ Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another ▪ Analyze how both repeating and growing patterns are generated	▪ Describe, extend, and make generalizations about geometric and numeric patterns ▪ Represent and analyze patterns and functions, using words, tables, and graphs
Represent and analyze mathematical situations and structures using algebraic symbols	▪ Illustrate general principles and properties of operations, such as commutativity, using specific numbers ▪ Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations	▪ Identify such properties as communtativity, associativity, and distributivity and use them to compute with whole numbers ▪ Represent the idea of a variable as an unknown quantity using a letter or a symbol ▪ Express mathematical relationships using equations
Use mathematical models to represent and understand quantitative relationships	▪ Model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols	▪ Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions
Analyze change in various contexts	▪ Describe qualitative change, such as a student's growing taller ▪ Describe quantitative change, such as a students' growing two inches in one year	▪ Investigate how a change in one variable related to a change in a second variable ▪ Identify and describe situations with constant or varying rates of change and compare them

Geometry		
Standard Instructional programs from pre-kindergarten through grade 12 should enable all students to—	**Pre-K–2 Expectations** In pre-kindergarten through grade 2 all students should—	**Grades 3–5 Expectations** In grades 3–5 all students should—
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	▪ Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes ▪ Describe attributes and parts of two- and three-dimensional shapes ▪ Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes	▪ Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes ▪ Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids ▪ Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes ▪ Explore congruence and similarity ▪ Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	▪ Describe, name, and interpret relative positions in space and apply ideas about relative position ▪ Describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance ▪ Find and name locations with simple relationship such as “near to” and in coordinate systems such as maps	▪ Describe location and movement using common language and geometric vocabulary ▪ Make and use coordinate systems to specify locations and to describe path ▪ Find the distance between points along horizontal and vertical lines of a coordinate system
Apply transformations and use symmetry to analyze mathematical situations	▪ Recognize and apply slides, flips, and turns ▪ Recognize and create shapes that have symmetry	▪ Predict and describe the results of sliding, flipping, and turning two-dimensional shapes ▪ Describe a motion or a series of motions that will show that two shapes are congruent ▪ Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs
Use visualizations, spatial reasoning, and geometric modeling to solve problems	▪ Create mental images of geometric shapes using spatial memory and spatial visualization ▪ Recognize and represent shapes from different perspectives ▪ Relate ideas in geometry to ideas in number and measurement ▪ Recognize geometric shapes and structures in the environment and specify their location	▪ Build and draw geometric objects ▪ Create and describe mental images of objects, patterns, and paths ▪ Identify and build a three-dimensional object from two-dimensional representations of that object ▪ Identify and build a two-dimensional representation of a three-dimensional object ▪ use geometric models to solve problems in other areas of mathematics, such as number and measurement ▪ recognize geometric ideas and relationships and apply them to other disciplines and to problems that rise in the classroom or in everyday life

Measurement		
Standard Instructional programs from pre-kindergarten through grade 12 should enable all students to—	**Pre-K–2 Expectations** In pre-kindergarten through grade 2 all students should—	**Grades 3–5 Expectations** In grades 3–5 all students should—
Understand measurable attributes of objects and the units, systems, and processes of measurement	▪ Recognize the attributes of length, volume, weight, area, and time ▪ Compare and order objects according these attributes ▪ Understand how to measure using nonstandard and standard units ▪ Select an appropriate unit and tool for the attribute being measured	▪ Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate types of unit for measuring each attribute ▪ Understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems ▪ Carry out simple unit conversions, such as from centimeters to meters, within a system of measurement ▪ Understand that measurements are approximations and understand how differences in units affect precision ▪ Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way
Apply appropriate techniques, tools, and formulas to determine measurements	▪ Measure with multiple copies of units of the same size, such as paper clips laid end to end ▪ Use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meterstick ▪ Use tools to measure ▪ Develop common referents for measures to make comparisons and estimates	▪ Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes ▪ Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles ▪ Select and use benchmarks to estimate measurements ▪ Develop, understand, and use formulas to find the are of rectangles and related triangles and parallelograms ▪ Develop strategies to determine the surface areas and volumes of rectangular solids

Data Analysis and Probability		
Standard Instructional programs from pre-kindergarten through grade 12 should enable all students to—	**Pre-K–2 Expectations** In pre-kindergarten through grade 2 all students should—	**Grades 3–5 Expectations** In grades 3–5 all students should—
Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them	▪ Pose questions and gather data about themselves and their surroundings ▪ Sort and classify objects according to their attributes and organize data about the objects ▪ Represent data using concrete objects, pictures, and graphs	▪ Design investigations to address a question and consider how data-collection methods affect the nature of the data set ▪ Collect data using observations, surveys, and experiments ▪ Represent data using tables and graphs such as line plots, bar graphs, and line graphs ▪ Recognize the differences in representing categorical and numerical data
Select and use appropriate statistical methods to analyze data	▪ Describe parts of the data and the set of data as a whole to determine what the data show	▪ Describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed ▪ Use measures of center, focusing on the median, and understand what each does and does not indicate about the data set ▪ Compare different representations of the same data and evaluate how well each representation shows important aspects of the data
Develop and evaluate inferences and predictions that are based on data	▪ Discuss events related to students' experiences as likely or unlikely	▪ Propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusion or predictions
Understand and apply basic concepts of probability		▪ Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible ▪ Predict the probability of outcomes of simple experiments and test the predictions ▪ Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1

Connecting Widely-Held Expectations with Mathematics Standards and Benchmarks

This work is the result of several rural school districts working to align the Primary Program's Widely-Held Expectations to the McREL Compendium standards and benchmarks.

(Kendall, J. S. & Marzano, R. J. (1997). *Content knowledge: A compendium of standards and benchmarks for K-12 education (2nd ed.).* Alexandria, VA: Association for Supervision and Curriculum Development and Mid-continent Regional Educational Laboratory, Inc.)

1. Understands and applies basic and advanced properties of the concepts of numbers

B-3	3–5	5–7	7–9	9-11	11–13
• Begin to recognize "one" and "more than one" • Count to nursery rhymes or the alphabet song	• Recognize and count up to five	• Count first by starting back at 1 each time something is added • Begin to pick up counting where they left off, starting with 7 and counting on to 9 • Count all types of things; play with counting forward or backward • Enjoy counting to 10 and idea of big numbers	• Begin to enjoy counting puzzles and games where they need to find a number between 10 and 99 • Order numbers from 0-10, and then 10-100 and much higher	• Begin to extend number sequences to take in large numbers from 1,000 to 10,000 and beyond	
		▪ Work with simple number facts showing different sums with many types of materials	• Represent more addition and subtraction "facts" in a variety of ways		
	• May recognize that two is always two and three is always three but does not apply this concept beyond five	• Begin to recognize that 10 is 10 or 20 is 20, no matter how objects are arranged in a group	• Make simple explorations with the concept of place value (combining groups of 100's, 10's, and 1's to make different numbers)	• Begin to understand the number system as a system built on tens particularly when working with base 10 blocks and other activities	• Begin to explore other ways to build number systems; for example, to think about numbers being represented by 0's and 1's
					• Begin to explore more complex number relationships and represent ideas in a greater number of ways • Begin to explore different simple number sequences which require more than simple addition and subtraction for their extension (2, 4, 8, 16, __ or 1, 3, 6, 10, __)

1. Understands and applies basic and advanced properties of the concepts of numbers, continued

B-3	3–5	5–7	7–9	9-11	11–13
• May use simple quantity words such as “one more cookie” or “more milk”	• Match objects in one set to objects in a second set				
	• Line up two or three objects using size or some other category	• Sequence things from the biggest to the smallest by size or other variable • May insert items into a sequence at the appropriate place	• May order things in a sequence in one set in relation to a sequence in a second set		
		• May isolate a set from a collection	• Are able to identify sets of objects with 2 or 3 attributes in common (separate triangles by color, size, and thickness)		
		• May realize that a collection can be sorted in more than one way	• Group numbers by twos, threes, fives, tens, and so on		
			• Begin to develop part and whole relationships and understand subtraction by separating a whole into parts	• Have a better coordination of parts and whole as related to both time and fractional concepts	• May develop the idea that the whole equal to the sum of its parts as a basis for the idea of percent (interest rates in savings accounts, cost of sale items 25%) • May begin to see the relationships between fractions and decimals

2. Uses basic and advanced procedures while performing the processes of computation

B-3	3–5	5–7	7–9	9-11	11–13
• Count to nursery rhymes or the alphabet song	• Recognize and count up to five	• Work with simple number facts showing different sums with many types of materials	• Represent more addition and subtraction "facts" in a variety of ways • Begin to develop part and whole relationships • Understand subtraction by separating a whole into parts	• Refine abilities to estimate • Build models of numbers 100, 1,000, 10,000 • Work on many whole number problems	• Begin to explore different simple number sequences which require more than simple addition and subtraction for their extension (2, 4, 8, 16, __ or 1, 3, 6, 10, __) • May begin to see the relationships between fractions and decimals • May develop the idea that the whole is equal to the sum of its parts as a basis for the idea of percent (interest rates in savings accounts, cost of sale items 25%)

3. Understands and applies basic and advanced properties of the concepts of measurement

B-3	3–5	5–7	7–9	9-11	11–13
• May follow the "path" of an object	• Use measurement words ("big and small," "short and tall," "near and far") • Line up two or three objects using size or some other category	• Sequence things from the biggest to the smallest by size or other variable • May insert items into a sequence at the appropriate place			
	• Identify portions when sharing	• Try measuring all sorts of things but with non-standard units	• Explore size relationships • Use rulers and yard sticks to measure length • Begin to estimate and measure and to use standard units to communicate similarities and differences	• Begin to see the need for a special measure • Continue to work on everyday problems involving length and may extend this to area, perimeter problems, using a variety of units • Begin to coordinate vertical and horizontal lines to help with ideas of area • May work on practical problems involving length, capacity, time and large numbers	• Use a variety of measurement tools • Begin to experience the ideas of mass and volume • Begin to use standard units for finding mass and volume based on many concrete activities

4. Understands and applies basic and advanced properties of the concepts of geometry

B-3	3–5	5–7	7–9	9-11	11–13
• May follow the "path" "of an object	• May follow the "path" of an object	• Begin to develop a stable idea of a straight line	• Begin to develop the idea of vertical and horizontal lines		
• Begin to pick out one thing from a group. Sometimes find two or three that are the "same"	• Sort using single attribute	• Classify objects in a variety of ways • May insert items into a sequence at the appropriate place	• Begin to classify things in more complex ways and use general categories and subcategories • May order things in a sequence in one set in relation to a sequence in a second set		
• Begin to identify simple qualities of things like "soft" and "hot" and "cold"	• Learn more qualities of objects ("thick" and "thin")	• May realize that a collection can be sorted in more than 1 way			
	• Compare objects • Line up two or three objects using size or some other category • Recognize and name simple shapes (squares, circles, triangles)	• Sequence things from the biggest to the smallest by size or other variable		• Explore size relationships	
• Begin to get some ideas of how things are alike and how they are different	• Match pictures to actual shapes	• May isolate a set from a collection	• Are able to identify sets of objects with 2 or 3 attributes in common (separate triangles by color, size, and thickness)		
	• Use language to begin to get ideas about space and time ("next to," "on top of," "before," "after")		• Begin to estimate and measure and to use standard units to communicate similarities and differences		• May be curious about making drawings to scale
					• Begin to explore three-dimensional objects
	• Recognize simple patterns	• Use pattern blocks and other materials to make and extend patterns		• Explore patterns in numbers	

5. Understands and applies basic and advanced properties of the concepts of statistics, data analysis, and probability

B-3	3–5	5–7	7–9	9-11	11–13
• Begin to recognize “one” and “more than one”					
• Begin to pick out one thing from a group. Sometimes find two or three that are the “same”	• Sort using a single attribute	• Classify objects in a variety of ways • Sorts, classifies, and displays information using graphs (concrete, pictorial)	• Begin to classify things in more complex ways and use general categories and subcategories • Sorts, classifies, and displays information using graphs (pictorial and symbolic)	• Become comfortable using simple graphs to show relationships	• Begin to use line and pie graphs to represent information and explore relationships
• Begin to identify simple qualities of things like “soft” and “hard” or “hot” and “cold	• Learn more qualities of objects (“thick” and “thin”)	• Collects first-hand data by counting • May realize that a collection can be sorted in more than 1 way	• Collects first-hand data by counting and measuring • Extracts second-hand data from presented sources (e.g., tables, lists, visuals, printed resources...)		
• May use simple quantity words such as “one more cookie” or “more milk”					
• Begin to get some ideas of how things are alike and how they are different	• Match pictures to actual shapes				
		• Reads, discusses, and interprets displayed data	• Reads, discusses, and interprets displayed data • Solves problems involving diagrams, tables, graphs		

6. Understands and applies basic and advanced properties of functions and algebra

B-3	3–5	5–7	7–9	9-11	11–13
• Begin to recognize “one” and “more than one”	• Recognize simple patterns	• Use pattern blocks and other materials to make and extend patterns • Sequence things from the biggest to the smallest by size or other variable • May insert items into a sequence at the appropriate place • May realize that a collection can be sorted in more than one way • May isolate a set from a collection	• Begin to classify things in more complex ways and use general categories and subcategories • Group numbers by twos, threes, fives, tens, and so on • Are able to identify sets of objects with 2 or 3 attributes in common	• Explore patterns in number systems • Explore size relationships • Become comfortable using simple graphs to show relationships	
					• May have some early experiences with the idea of variable